Ordinary Differential Equations.

Higher order Linear Differential Equations.

The general form of a nth order linear differential equation with constant continual.

18 as day and dynamic tonstant and Remy Remy dynamic dyn

where as, as, as can are constants, as so.
and R(x) is a function of x.

there D=1 day, D2= d2 op live D2 = dn

(a) and Dy + a, Dry + fin + and py + any = Rin)

[and Dn + and Dn=1 (fint and D) + and = Rin)

GESSMETE (FUND) AT

Com where from and table to the

Complementary function:

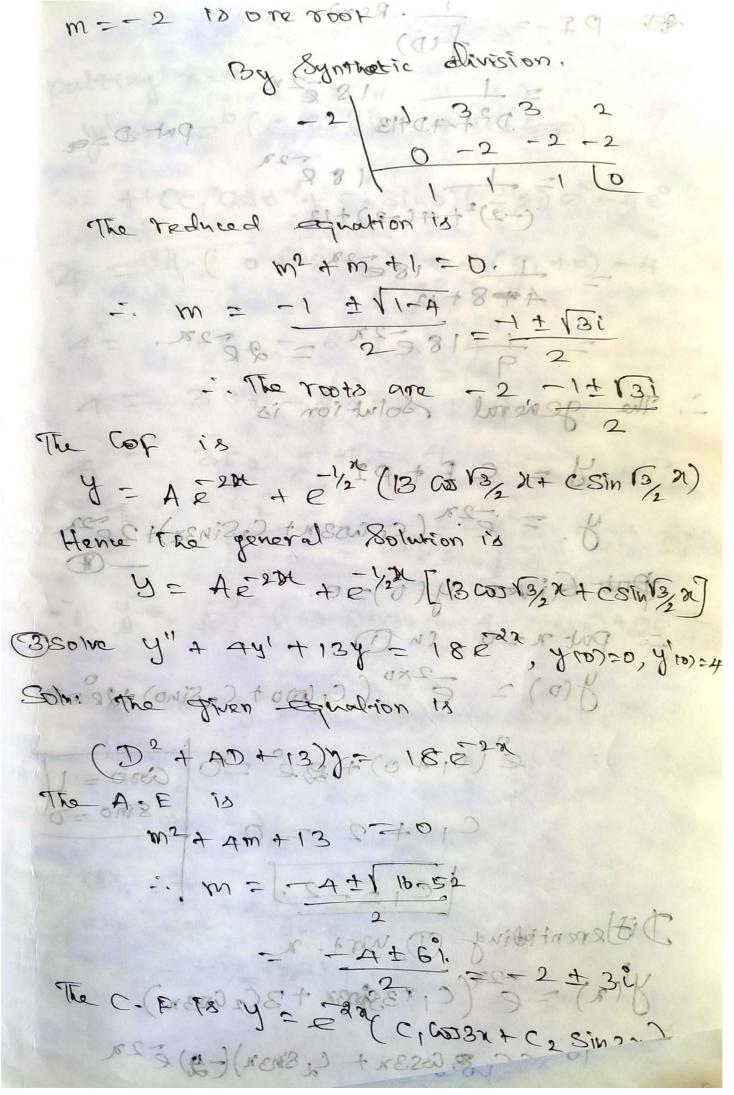
and distinct.

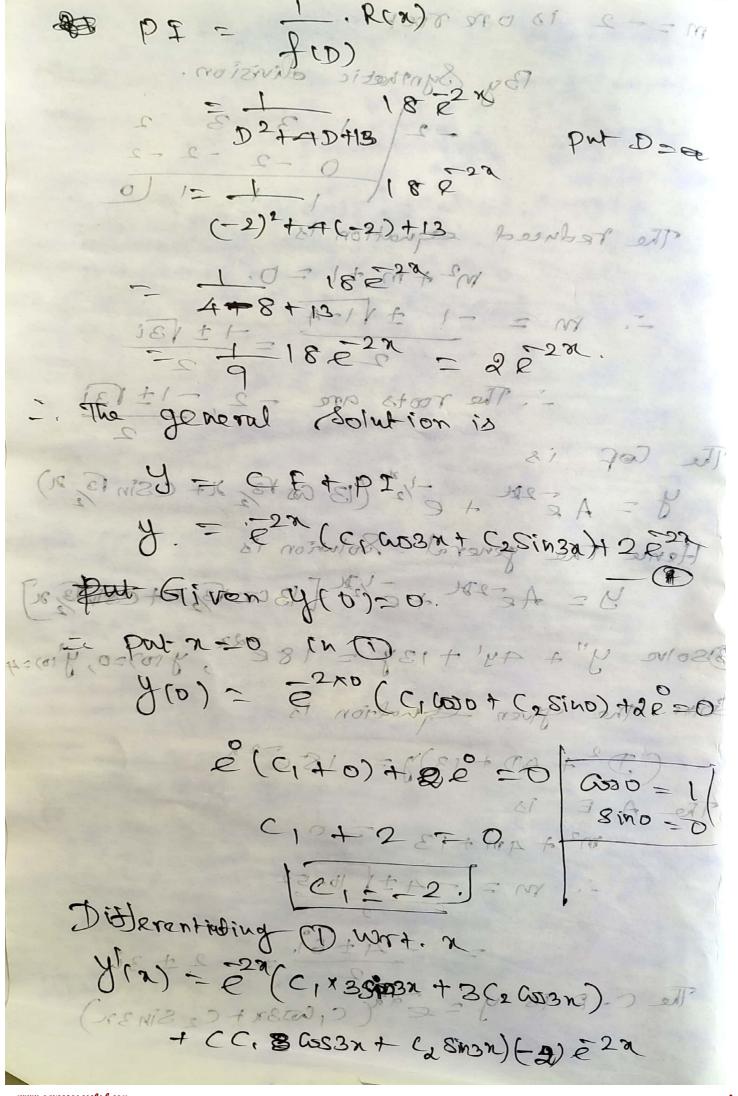
Then y = C12m1x + C22m2x + ... + Cnemnx.

Cade(2) All the roots of the A. F. are real and Some are equal.

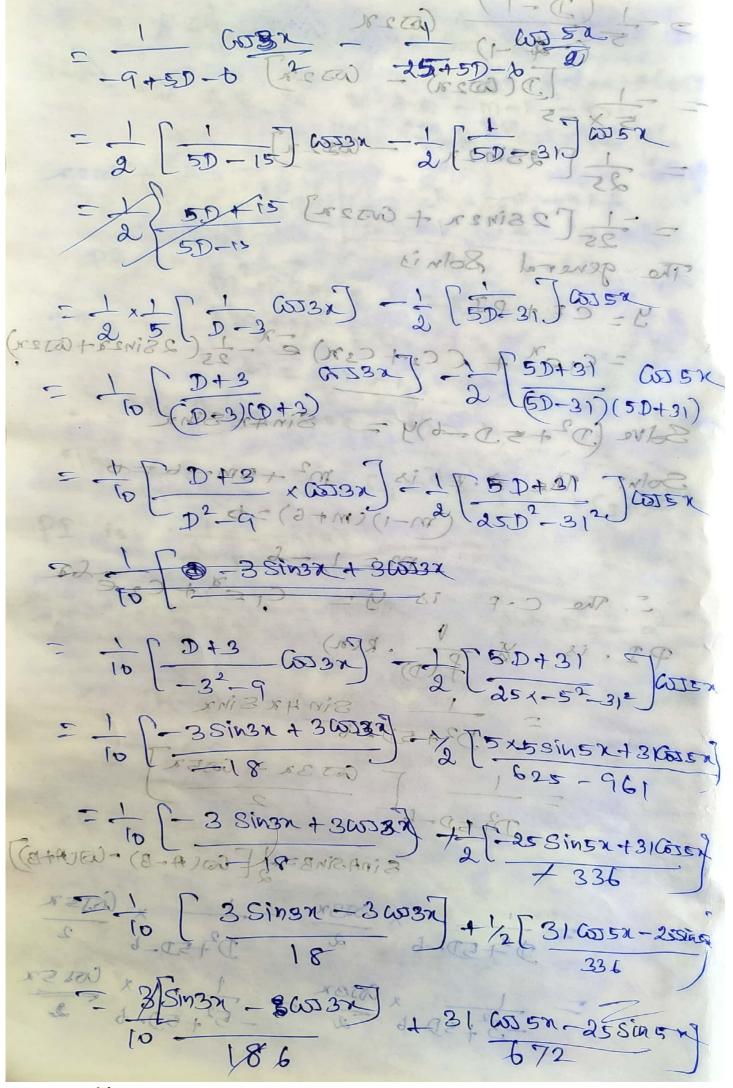
(ase 3) (CI+ C2 x + E3 x + ... + Cn x n-1) mx

Case 3 The Roots are Complen in the form of atiB. attendi-piedu (C' col Bx + C5 8in Bx) mentil prebro star of a mot loverep est Solve id y toisy por laist of the parties 119 = 180/no The diver- no beguation bis open staded Dy mes. (D20+D120) y = 6! (x The auxillary equation is apart D = m (212 = hub+ hard + w-5 + Aparis + has co 1) 83 4 (m-1) (m+2) (m+2) (m+2) (m+2) - The Complementary function CLAP + FERFE OP CMIN STONE MEN is) y = Aer + Be an proformelymo 080 (1) All The 20062 (2) The Solve dig + 3 dig + 3 dy and and 3 dy 200 mg Sofn: The Sten Equation is A (2) 2800 D3+3D2+3D+20) y=0000 6000 15 The A.F. is 15 m m3 + 3 m 2 + 3 m + 2 = 0 (3 93)





\$ - 4 8 2m = 10 - C+EC) N/02. pulting n=0 y'(0) = 00 (C, x3 Sino + 3(2600) + (CC, Osio + C2 sino) (-2) 20-40 4 = 01. (0 + 8 C2) # 2 (C1+0) - 4 4= 3C1-201-4 $4 = 3c_{2} - 2x - 2 - 466$ ett. 4 = 13C2 +4 - 4 3(4) = 23 C2 + 4 - 4 3(4) = 23 C2 + 4 - 4 C2 = (A13; 1) 21 29 - y = = 20 (-2 Coo3n + 43 8in3n) +2e2 Hw: 6 Solve y" + Ay + 3y= 8 2 76 22 given y(0)=0 and y'(0)=3. 4--21 = 3x + = = x + ex - 25 ex. dn2 + 4 dy + 134 = 2 = 1 4 = -1/5 22x (C1053x+C2 Siu2-) C055x (-- 52+5D-6



prob. 80/ve (D2-4D+A)4= 824 + 00122 Given (D= 4D+4)y = e2n+ GD2x The auxiliary equation is & Ig m2-4m +4 =0 r (m-2)2 = 0 + + CA - C C. F Yer-(An+C2), e2x PZ, = 10 10 x R(x) (16) 9.9 = = D2-40+4 A Replace Dby a = 12 12 14 P. 9 Trenista 2000 9-9 Exsis deeds Jugar = 2 . Trope 22 29+ 19+ 30. 20 + wolfwood

TERREL RECONS = 22. F"(D) e22 1000 7 83. - 98 F-+ CH - C.7 sw/08 . dord PT, = Me 2a. = B(++ C+ =C) reviles

PT, = 2 bi noidaupa prailinus elle 22-AD+A = 5 (5-11/2 x = R.p. $\frac{1}{2^2 - 4D + 4}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{2$ P pé C 25 P. P FS (Cida 2 = C -4-8it 40 = R.P 21 2122 - R.P - [assarisinan) = R.P.L. [Co22 - Sin2x] PP2 2 - 1/8 Bin 22. -. Solution M= CF +PI, +PI2

prob Solve (D2+6D+8)y= =2x+ co2x Given $(D^2+6D+8)y = \frac{-2x}{2x} + (1+ao)2x$ A.E. $m^2+6m+8 = 0000$ CIF A E 27 + BE TX = P AI NOS P.I renie 52+60+80 + 20 = Replace D by-2 $P_{2} = \frac{1}{2} + 10 + 8$ $P_{3} = \frac{1}{2} + 10 + 8$ $P_{3} = \frac{1}{2} + 10 + 8$ - R.P D2+6D+8 = R. P (21) 2 + 6 (2i) + 8 = R.P = A +12i+8 = R.P 1 +121° 222 R. P 45122 (4-12i)

= R. Po 4-12 = e12 x 02+ (C) 20108 don = (8+Cd+C). (20) = p(8+Cd+C). (20) P (R.P) 4-120 (Cos2n +isin2n) 1600 = 8+md+ m 3.A = Rep 1 [400122 + 12 Sin22]

160 2 600122 + 6 Sin22 Soln in 4 = CF + PI, + PI, + PI3 2-pd c orders 4- CF + Cot 2nd + 3 Sin2x 1. 4

-27 AP + 132 m + 1 + Cot 2nd + 3 Sin2x

-14 P + 132 m + 2 P + 1 + Cot 2nd + 3 Sin2x

-14 P + 132 m + 2 P + 1 + Cot 2nd + 3 Sin2x

-14 P + 132 m + 2 P + 1 + Cot 2nd + 3 Sin2x

-14 P + 132 m + 2 P + 1 + Cot 2nd + 3 Sin2x

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-14 P + 132 m + 2 P + 1 + Cot 2nd + 3 Sin2x

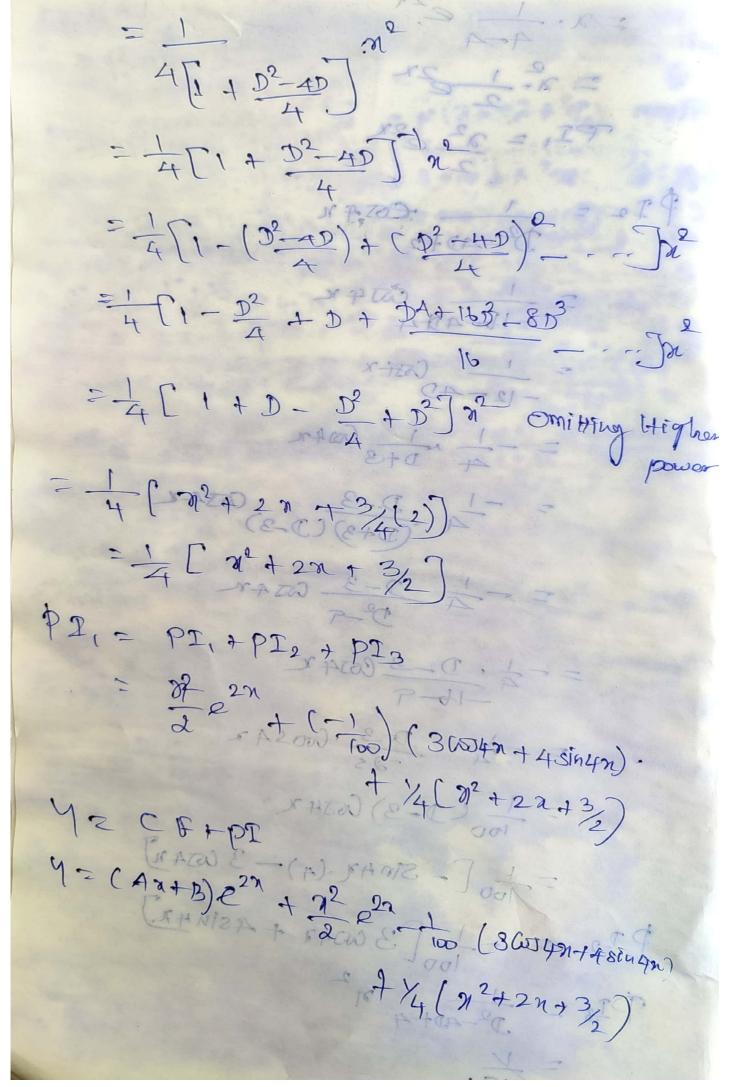
-14 P + 14 P + Edve 1 9-10 + 1)4 = 22 + Cos42 + 22 14 Solution: Given ()2-10+124 + 22 14 Cos42+212 The A-E is m2-Am+4=0. 803 (m2) = 0 C. F. = C. AafB) 22 3400 m - 2,2 PI, = 21600 + 2200 = 1+3c1+ e2x 9.9 sei – 22 Rule fails ej -1220- A+1

= x. 1 e29 = 2 - 1 2x (ax-a. + 1)x PI, = 220 POX SC F 17 1 \$70 = 1 - COSAN \$2 AD (2) + (CO-C) - 17 = = = 15-494 + C + C - 17 + pith puriting 10 10 Cost x

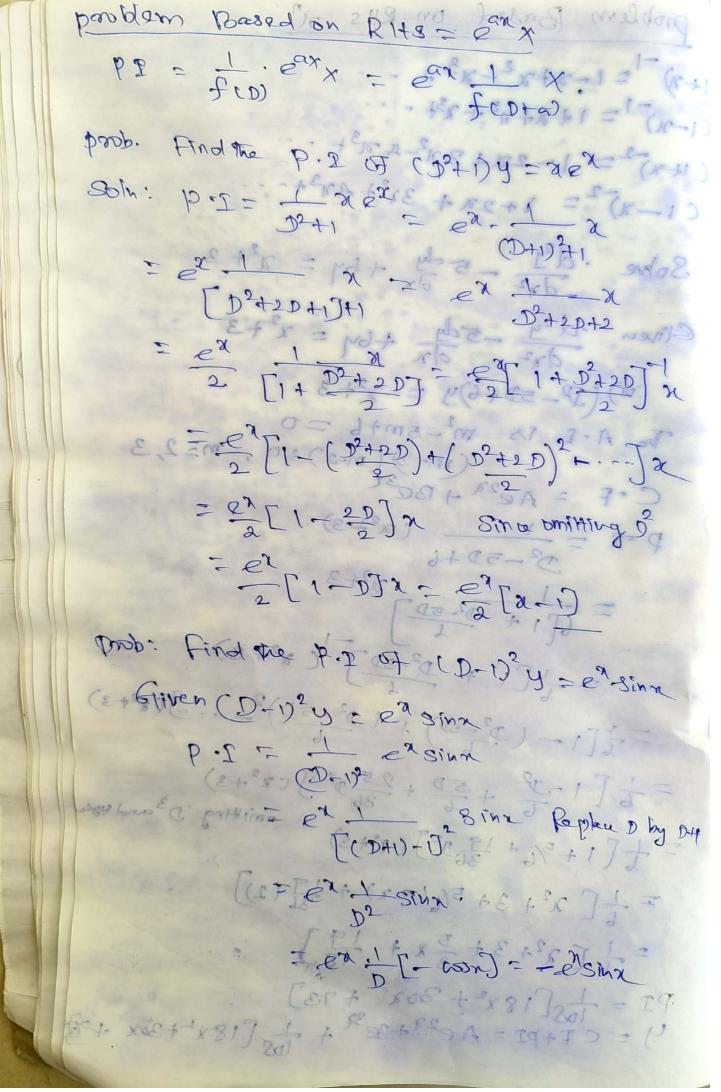
pith puriting 10 10 Cost x

- 12 5 40 60 - C + 1] = 3 = -1 (D-3 (D-3) = - 1 19-3 COJAN == =-4.D-3 BOJ4 2 19+,19 =,19 - GRANE D-3 : CNOSA2 6 (5 = 1 ()-3) C6542 = 100 [- Sin An (A) - 3 COSAN]

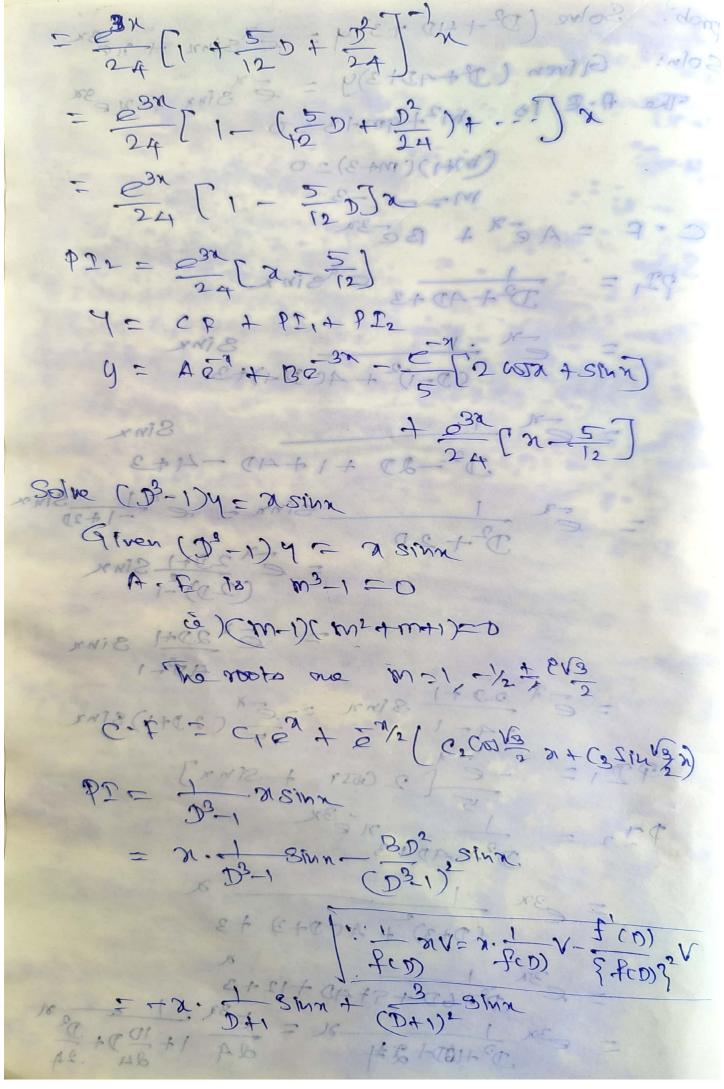
P22-100 [3 COSAN + ASIN 42] DI3 = D2-AD+ A 212



problem Based on RHS non bessel medung (1+n) = 1-x+x2-x3+ - - xx3. (4x) = 2x+3x2-43x3+ . q = 10117 . dosq Solve de dn2 = 5 dy + by = 22 + 30 Stren 43d 2y -5 dy +6y = x2+3 dx2 dx2 dx +6y = x2+3 of 22-50+6)y = x2+3+1] s The A-B 18 m²-5m+6 = 0 C-F = Ae2d + Be2x + Be2x per 1 = 52 - 50 +6 (2 +3) -1] = = $= \frac{1}{6[1+10^{2}-5D]}$ 112 P3 24 50 7. 9(x24 3) 7 . domp $=\frac{1}{2}\left[1-\left(\frac{D^{2}-5D}{b}\right)+\left(\frac{D^{2}-5D}{b}\right)^{2}\right]...\left[\left(\frac{8^{2}+3}{b^{2}}\right)$ $=\frac{1}{6}\left[1-\frac{12}{36}+\frac{50}{6}+\frac{257}{36}0^{27}\right](2^{2}+3)$ $=\frac{1}{6}\left[1+\frac{50}{6}+\frac{19}{36}0^{27}\right](2^{2}+3)$ omitting omitting D3 and Hola = 1 [22 + 3 + 5 (6 (2x) + 13 (2)] = 1 [22+3+ 5x+ 19] PI = tos[18x2+30x +73]
Y = CF+PI = A e2x+Be3x + tos[18x2+30x +3]

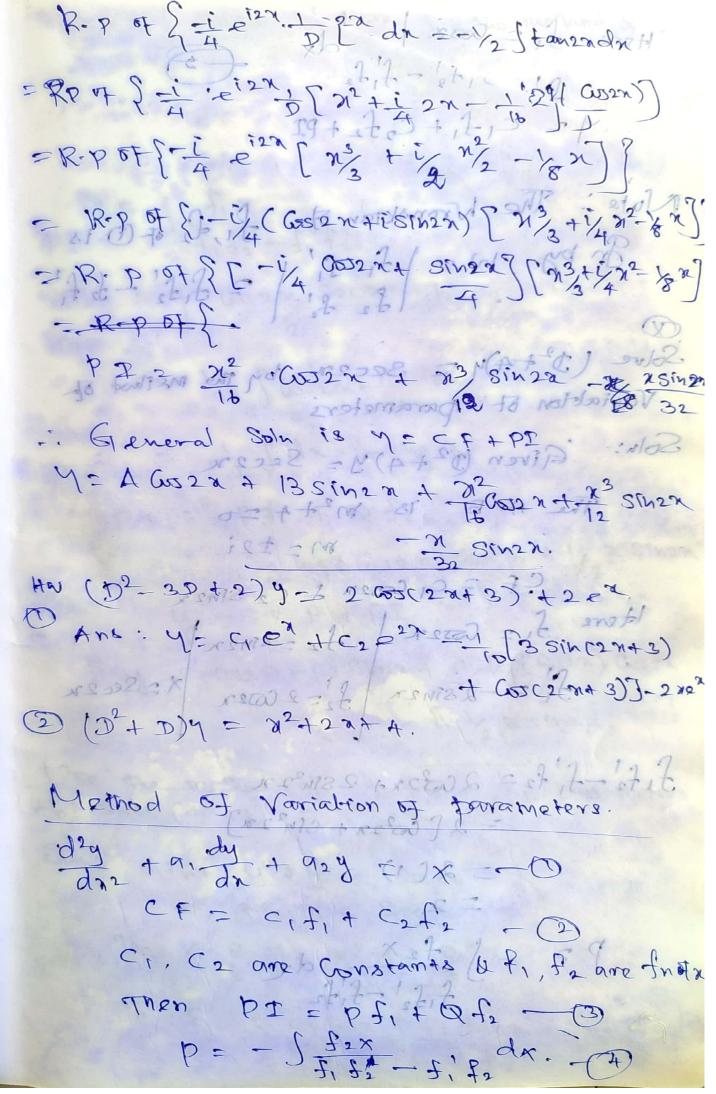


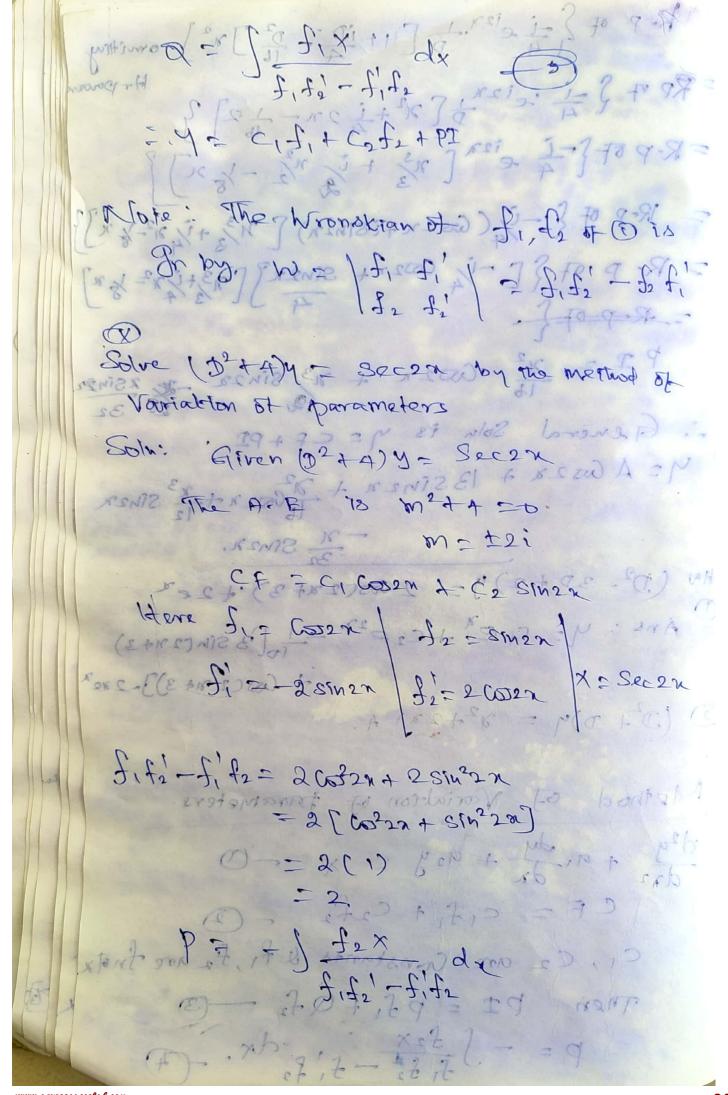
probi. Solve (D2+4D+3)4 = Exsinn + ne Soln: Gilven (D+AD+3)y = = = 8 lux + 81 = The A. E 18 m2+ Am +3 =0 ... (m+1)(m+3)=0 p1 = D2+AD+3 = 3 in 2 = THAT A SOU COD-12 + ACD-1743 [=== N 180 + 1 + 4D - 4+3 5=(Horry 1/1-1/2)-1/20+1 $D^{2} + AD + 3$ $= 8^{3} \times 1$ $= CD + 3^{3} + ACD + 3 + 3$ $= 8^{3} \times 1$ $= 8^{3} \times$



5-50 x 20-1 Sinn + 52+20+1 =- N D-1 8 mx + 3 8 mm = 21 [D(SMA) FBMA) +3[SSINAda] PI = 2 Com-sina) - 3 sosn The general Solm is y = CF + PIThe peneral Solm is y = CF + PI $y = C_1 e^{2} + e^{2} +$ 13++ 2 (Con-3mx) -3/2 Com #.10 Solve (D2+50+A) 48= E28in22 Ans: Ac + Bean - en [3002x + 25 m2x] and The P. P. Of (D2+1) y= Next Solution Description Descript 1. C. C. = expline p2+20] 2 (x (2) + (D2+2D) + (D2+2D) 2 ---) x

= = [1-20]x constituy the powers = er Ede Dat Mile 1-C. K Chordel Excand Crmescol & 1202 - 10 - 100 devising 86/2 19 N= CL+ 61 Solve to 2 quation (D2+ 4)4 = 22 Coses The A = E 12 m2+4=0 MED LE TOO trave m = 4 c 2 C- F- A C55220 +18 SIW2XC2+50) 20/08 07. Crapper Pot 73012x DA: ANA = REPR CH (D+12)2+4 9 ent bord R. P Ri29: 1 [1+ D2] 72 = R-P = [22] [1+ iD] n2 = R.P 2122 1+ 10 - 52 - 103 J 82 5 R-P 07- (-12x 1 [1+ 10 - 12 - 103] x2 2



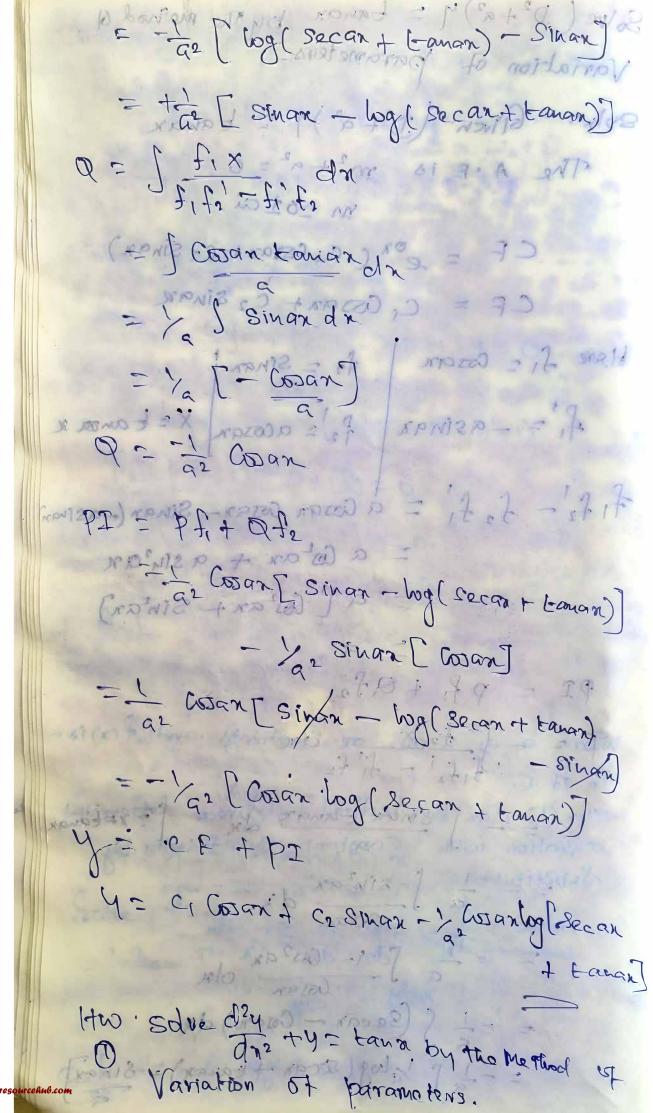


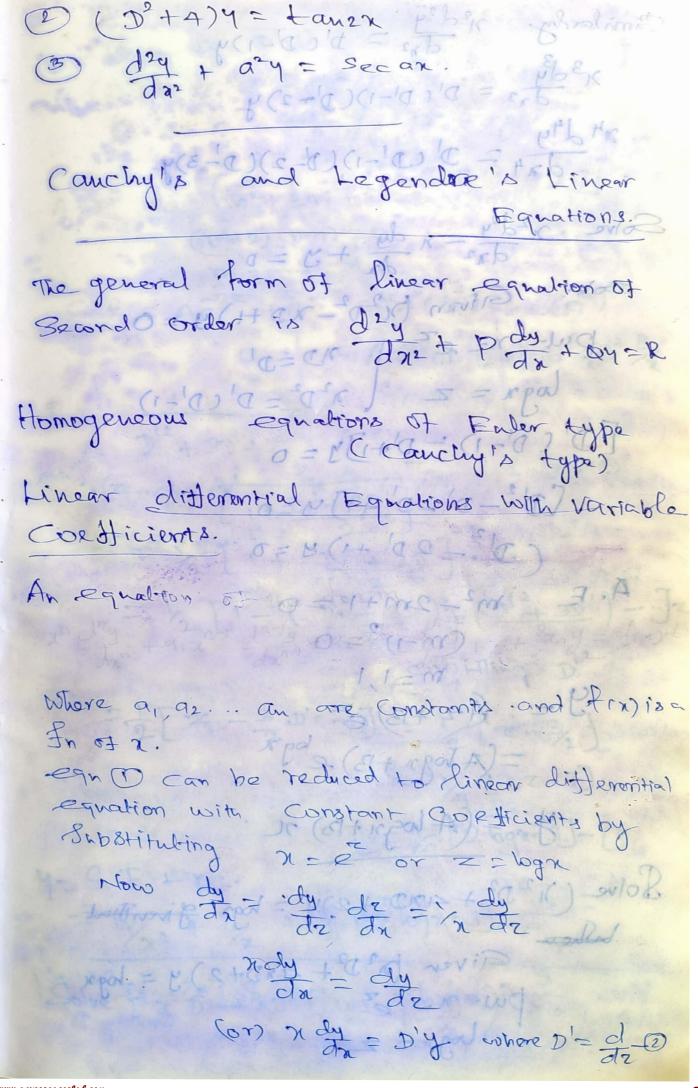
= - Sin 2n Sec22 da == 1/2 Stanzada soon 2 x 2 and of the control control (= 1 NIE + x 200 1= 1 (cos 2n) Q= J fix fifi'-fifi da x it = 1 Goz 2 x Sec 2 x dz = 1/2 J G032 x (632 x) = - Jdxxb=rygx P2 = P\$, + Q \$200] pal = /2 log (Cos 2 20) Cos 2 2 4 1/2 21 Sin 22 y = cffpf-letit = 4 = C (0522 + C2 Sin22 7 1 log [20122) Cot22 t /2 a sinex. Use the method of Variation of parameters to solve (D2+1) y = secon Soln: Giren y"+ y = secre 5) (D2+1) y = Secx 520 = MARK + Craw par Frants metto 7. Sera 19 3 M C - F = C1 Com + C2 8 inx = 0151 + 0252

Here of = Coox of for Sinc (mai fazzan de com fife = of 2 fi + cos2x + sin2x =) = - Sinx Secz dz E - J. Sinx dx dx = - Jeannarby = log [Cours] p + 759 CM12 16 1/2 1 State 2 3 3 4 1/2 3 3 5 1/2 3 3 5 1/2 3 5 1/2 3 5 1/2 MED (NEED) COUNTY SECNED OF MED OF ME - da = x pfitQf2 considered = lug [Cosas Cosa + x sinx] god = = OFX pod (COIX) + x ZINX 4 = C, Cosn + C2 Shx + Cosn log(con) + 215mg KNIZ OF KOUD

Solve (D2+a2) y = tanon Variation of parameters. solver of Det at) we Fanax The A.E is mit a = 6 it m sotai of t CF = en (CC (CS) STNAN) CF = C, Copan + C2 Sinax Hame Siz Cosan fiz Sinan Ai = - asinax fi's acosan X = tanoax fifi-fifi = a Goran Goran - Sinan (- asimon = a Cotan + a sturan

(round + moss) por a per Coran + Sinan) Imacol Jan 18 PI = PR + Qf2 General Singer - Marie Juscos 1, fil - fifz Comment of Sman Lanan du estation de la constant de la consta a J-1- Co2an da = - 1 S (3ecan - Cosan) dass with = - - [Valog (socan + teanan)



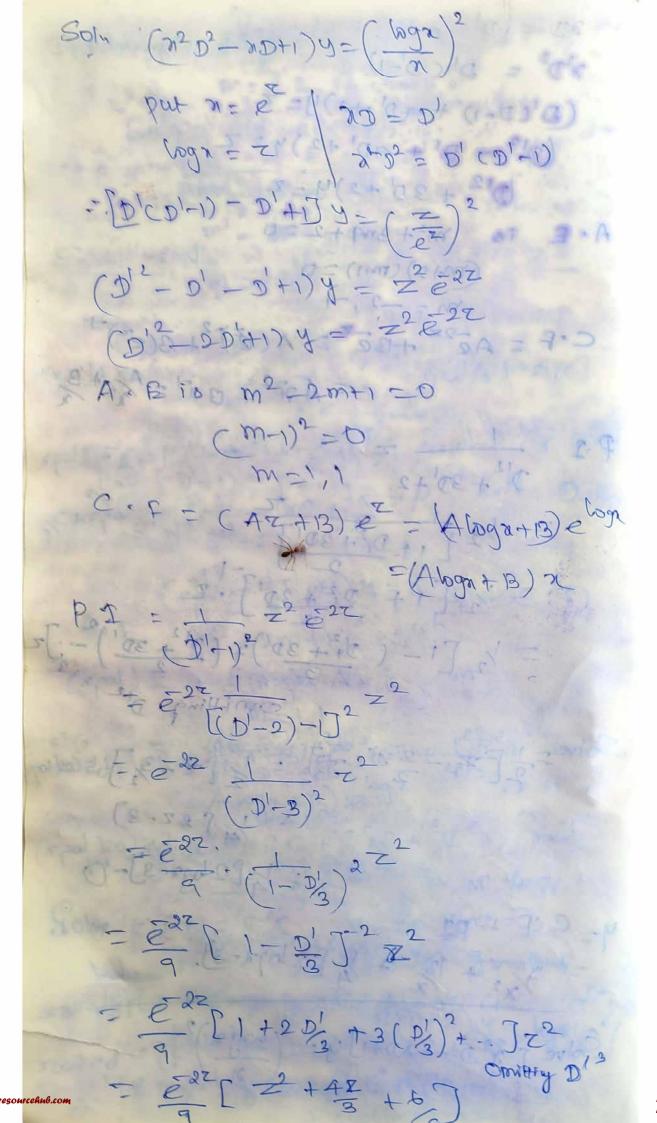


Stanilarly 12d2y - D'(D'-1)y

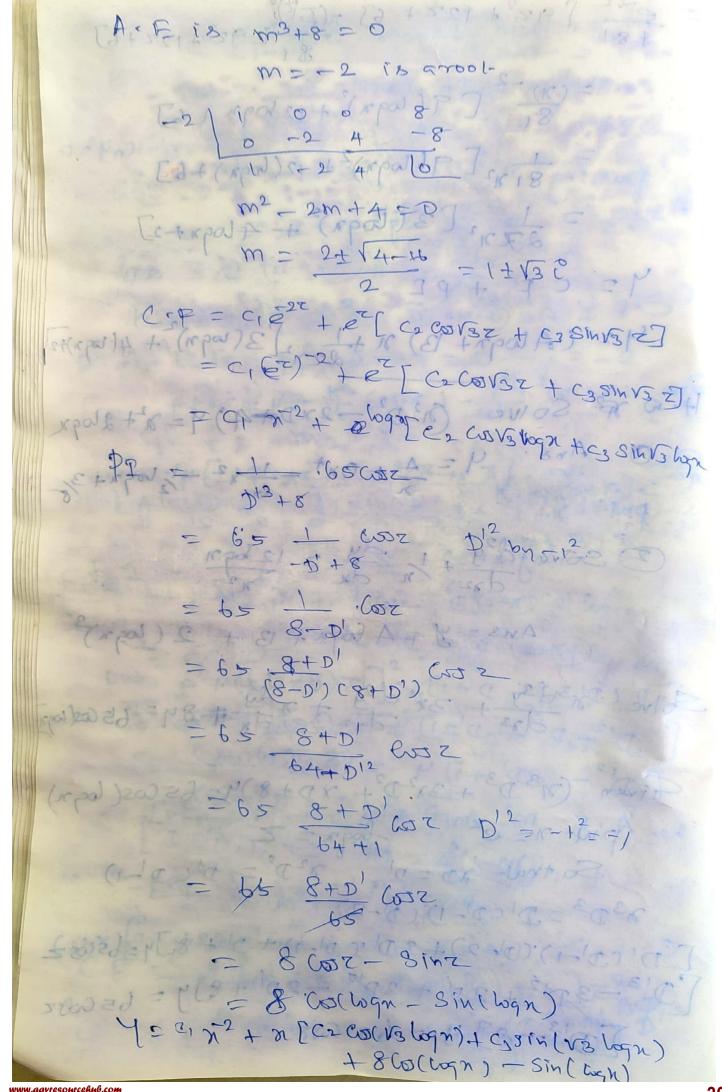
3dy

dr3 = D'(D'-1)(D'-2)y at dty da4 = D'(D'-1)(D-2)(D-3)/prismo) Some $\chi^2 d^2y - \chi dy + y = 0$ 30h Gilven (1202-20 H) 4 = 0 $pul-n=e^{z}$ $\log n=z$ n = D' $\log n=z$ [D'(D'-1) = 0/+]y = 0 observed [Di2-Di41] & Somme (Silver) (D12-201+1) y=0 Ampisito 200) A. $E m^2 - 2m + 1 = 0$ $(m-1)^2 = 0$ 201(11-47-13) E = (A hogn +13) e bgn y = (A logn+13) 2 Solve (h2D2 + 4nD + 2)y = bgn given that Siven (n2 D2 + 4nD+2) y = hogn logn = Z

26



= ext [922+ 122+ 6] = (E) [922+122+6] = (N) -2 [9[logn) 2+12 logn +6] - type [9(logn)2+ 12(logn) +6] 2772 [3(bga)2 + 4bgn+2] Y = CF + PE 4= (A wgn+ B) n + 1 22n2[3(wgn)2+ 4f vgn)72 (2) D2 - 2 ND + 4) 7 = 2+ 2 logx 02 And: 4 = A + Bxt - 1/2 logx + 3/8 Ans: y = A Cogn + 13 + 2 (logn } Solve n3 d3y + 3n2 d2y + n dy + 84 = 65 Cool Logs Given (N3 D3 + 322 D2 + xD+8)4=65 cos(bgn) put n= ez, logn= Z So that aD = D' $a^2D^2 = D'CD'-1)$ 23D3=D1cD1-1)(D1-2) [D'(D'-1)(D'-2)+3D'(D'-1)+D'+8Jy=65052 1)13-3pt2+2pt+3pt2-2pt+8Jy=656002 D13+8] 4 = 65 602



Legendre's Linear D.E An equation of the form. (antb)h dry + c, (antb)n-1 dry + -+ cny =0

putting antb = et dnn-1 + -+ cny =0 Z = log(gn+b) (antb) D = aD'(com) E(emb) D = aD'Salve $(x+2)^2 \frac{d^2g}{dn^2} - (n+2) \frac{dy}{dn} + y = 3n+4$ 9+ 19+ 75 -N Soln: Gren [(x+2)2D2-(x+2)D+1] y=3x+4 s-(pm-1911) = te (o+m) to + (e+m) for A JEN log(x+2) = Z [4+10 pa]] to O1+2) bit=16(1+10)] who (2+2)2D=D(6D-1)C(1+1) [D'(D'-1) - D'+ Jy = 3(25-2) + 4 tog [D12-D1-D1+1)y=13e2-6+4 [D'2- 2D'+174 = 132722 The A = E m2 - 2m+ (= 100 0 = 50 (1+x) (M-1)= OF + (1-10) 07 m = 1/2 7 4 A = (AT+B) = = [A log(2+2)+B(X+2) PI [= (7)-1) 128+ CUAR 190 A

Keldace = Z.] 3en of B northwy 3 (A + MS) = Z² 1 3e⁷ = E 3e² = 3 2e⁷ CO = C (HAMP) = = 3 [log(x+2)]² (x+2) $0 p_{12} = \frac{1}{2[-2e^{0}]} = -2\frac{1}{(-1)^2} = -2$ 4= CF + P7, + P82 4=[A log(N+2) + Pof(N+2) + 3[log(N+2]2/N+2)-2 5 = (C+x) pal Solve · [(x+1)2D2+(n+1)D+ []= 4 Cos[Cogen+1)] Solv. [(x+1)2D+ ca+1)D+Jy = A Cot logenty) Z = log(1+x) + fine = 10 fig. (2+1) D = D1 P (7+ 1/28 - 1/2) (x+1)2D2 = D'(D'-1) [D'CD'-1) + D'+ DY = 4[Coo 2] AB is matter F = CAT + BOST = MARQUELEN BOX (X+1) CF = A COZ + B Sinz = Acos[bg(x+1)]+Bsin[bg(x+1)]

= 4 · D12+1 COZ Raplace D'Eby-12 ्राच्या व = 42 20 coz = 22. 1 coz z 0 = + = 2Z S TOTAT A = 72Sinz = 2 log(1+x) 8 in [loge 1+2) Y=A cos[log(a+1)]+B 8 in[log(n+1)]+2log(a+1)x Sim [Gg(xH)] Sdr (3x+2) = = + 3(3x+2) dy - 364 = 3x+4x+1 Sola. Given $[(3x+2)^2]_{32}^{24} + 3(3x+2)D - 36Jy = 3x^2+4x+4$ put 3×+2= e (og(32+2)=7 32-2-2 (= 3e - 2/3 24-(3x+2)D=30 $(3x+2)^2D^2 = 90'(D'-1)$ 9 D' (D-1) + 3 (3 D')-36) y = 3 [13ex-2] + 4 [3ex -2] +

[90'-90'+90'-36]y=3['ge+4-4e7] (9012-36)y = 3e2+ 4, -4, e2+ 5e-8, 4 = 13 22 - 43 +1 = 13 22 -13 = 9 = 12 -4) y = 1 = 27 = 27 - 27 A-E m2 4 = 0 7 9 3 7 5 M 2 5 4 C.F = A 2 + B = 2 = A (2) + B (2) -2 *(H) pale = (4 (3x+2)2+13 (3x+2) 22 A2 N 10 Di2 - 1 27 Replace D'bys $= \frac{Z}{108} \left(\frac{2}{e^2} \right)^2 = \frac{Z}{108} \left(\frac{3}{108} \right)^2$ [8 2 18 4 (8 - (10 8) 10 F (13 n+2)²

